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Overview



Introduction

- Constitutive Modeling of Soft Tissue
 - Discrete Models (Particles + Spring Mass Models)
 - Continuum Models (Hyperelasticity)
- Measuring Soft Tissue Deformation
- Discretization Methods
- Example of Soft Tissue Models

Soft Tissue Modelling



Important for :

- Creating Physiological Models of the human body (understanding / teaching)
- Image Registration (deformation prior)
- Diagnosis : finding pathological behavior
- Therapy planning
- Procedure simulation

Computational Models of the Human Body



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Soft Tissue Deformation



- Objectives of a soft tissue models :
 - Reproductive :
 - Able to produce simulation comparable to observations
 - Discriminative :
 - Associated parameters used to cluster a given population
 - Predictive :
 - Predict behavior under specific conditions

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Discrete Constitutive Models

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- Models defined on a set of points
 - Particles : material points in interaction
 - One particle interacts with all other particles (no fixed neighborhood)
 - Use of an interaction potential based on :
 - Distance between 2 particles
 - Angles between 3 particles
 - Angles between 4 particles
 - Link with molecular dynamics



Source http://www.ch.embnet.org/MD_tutorial/pages/MD.Part2.html

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Discrete Constitutive Models

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- Models defined on a set of points
 - Particles :
 - Example of potential : lennard-jones



Discrete Constitutive Models



- Models defined on a set of points
 - Springs :
 - Based on a set of edges (fixed topology)
 - Main idea : applied force is proportional on the variation of edge length :



1D Spring-mass system



Dynamic problem: What is motion of the mass when acted by an external force or is initially displaced?



y(t) = Spring extension dre Anaromical Human project http://3dah.miralab.unige.ch



Newton's Second Law of Motion

the acceleration of an object due to an applied force is in the direction of the force and given by:

$$F_{net} = ma = my''(t)$$

For our spring-mass system

$$my''(t) = \underbrace{F(t) - cy'(t) - ky(t)}_{F_{net}}$$

$$my''(t) + cy'(t) + ky(t) = F(t)$$

Undamped Free Vibrations





$$T = \frac{2\pi}{\omega_0} = 2\pi \left(\frac{m}{k}\right)^{1/2}$$
$$\omega_0 = \sqrt{\frac{k}{m}}$$

Period of motion

Natural frequency of the vibration

$$R = \sqrt{A^2 + B^2}$$

Amplitude (constant in time)

Damped Free Vibrations



$$my''(t) + cy'(t) + ky(t) = F(t)$$

$$my''(t) + cy'(t) + ky(t) = 0$$
no external force
Assume an exponential solutio $y(t) = e^{rt}$
Then
$$y'(t) = re^{rt}, \quad y''(t) = r^2 e^{rt}$$

and substituting in equation above, we have

$$mr^2 + cr + k = 0$$

(characteristic equation)

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Damped Free Vibrations



$$mr^2 + cr + k = 0$$

Solutions to characteristic equation:

$$r_1, r_2 = \frac{-c \pm \sqrt{c^2 - 4km}}{2m} = \frac{c}{2m} \left(-1 \pm \sqrt{1 - \frac{4km}{c^2}} \right)$$

$$\begin{split} c^2 - 4km > 0, \quad y = Ae^{r_1t} + Be^{r_2t} & \text{overdamped} \\ c^2 - 4km = 0, \quad y = (A + Bt)e^{-ct/2m} & \text{critically damped} \\ c^2 - 4km < 0, \quad y = e^{-ct/2m}(A\cos\mu t + B\sin\mu t) \\ & \text{underdamped} \end{split}$$

3D Spring-Mass



Point Masses linked with springs



$$W = \sum_{j \in N(\mathbf{P}_i)} k_{ij} \left(\left\| \mathbf{P}_i \mathbf{P}_j - l_{ij}^0 \right\| \right)^2$$

$$\mathbf{F}_{i} = -\frac{\partial W}{\partial \mathbf{P}_{i}} = \sum_{j \in N(\mathbf{P}_{i})} k_{ij} (\|\mathbf{P}_{i} \mathbf{P}_{j}\| - l_{ij}^{0}) \frac{\mathbf{P}_{i} \mathbf{P}_{j}}{\|\mathbf{P}_{i} \mathbf{P}_{j}\|}$$

Spring-Mass Advantages

 Use Point Mechanics instead of Continuum Mechanics

$$m_i \frac{d^2 \boldsymbol{P}_i}{dt^2} = \gamma_i \frac{d\boldsymbol{P}_i}{dt} + \boldsymbol{F}_i$$

- Easy to implement :
 - Only needs a graph
- Invariant to rigid body motion
- Widely used in computer graphics

WinSpringies

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Spring-Mass limitations



- Behavior depends on mesh topology
 - No formal link between topology and behavior (except for simplicial complexes)
 - Difficult to link spring stiffness to physical parameters (Young Modulus, Poisson Coefficient)
 - Use of Neural Network or genetic algorithms to identify stiffness parameters
 - Weak behavior for shear stress

Shear-Stress behavior

- Weak shear-stress
 - See X. Provot paper





(b) After 200 iterations

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Continuum Mechanics





1D Elasticity





Point X is deformed into point $\phi(X)$

How much deformation around point X?

1D Elasticity : stretch ratio





Rest length : 2 dx New length : $\phi(x+dx) - \phi(x-dx)$

Stretch ratio at X is
$$s(X) = \frac{d\phi}{dX}$$

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1D Elasticity : strain energy



What is the energy necessary to deform the bar ?

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Deformation energy W depends "how stretched" the bar is



W depends on strain ε = distance between s and 1

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1D Elasticity : strain





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1D Elasticity : stress



Stress is the energy conjugate of strain

 $\frac{\partial W}{\partial \sigma}$

$$\sigma = \frac{\partial W}{\partial \varepsilon} \qquad \varepsilon =$$

Extensive	Intensive	
Variable	Variable	
Position	Force	
Angle	Torque	
Volume	Pressure	
Strain	Stress	

- For $\alpha = 1$ (First Piola-Kirchhoff) nominal stress
- For $\alpha = 2$ Second Piola-Kirchhoff stress
- For $\alpha = 0$ Cauchy stress

St Venant Kirchhoff Material

Basic Material :

W is a quadratic function of strain

Stress is proportional to strain

• **1D case** : λ is the material stiffness

$$W = \int_{\Omega} \frac{1}{2} \sigma \varepsilon = \int_{\Omega} \frac{\lambda A}{2\alpha^2} \left(\left(\frac{d\phi}{dX} \right)^{\alpha} - 1 \right)^2 dX$$

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 ∂W

1D Elasticity : discretization

Represent the bar with a single segment



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1D Elasticity : discretization

Represent the bar with a single segment



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3D Elasticity



- Deformation Function
- $X \in \Omega \mapsto \phi(X) \in \Re^3$ • Displacement Function $U(X) = \phi(X) - X$



Deformation Gradient

The local deformation is captured by the deformation gradient :



Stretch Tensor

 $C = \nabla \phi^T \nabla \phi$

Distance between point may not be preserved



Distance between deformed points

$$(ds)^{2} = \left\| \phi(X + dX) - \phi(X) \right\|^{2} \approx dX^{T} \left(\nabla \phi^{T} \nabla \phi \right) dX$$

Right Cauchy-Green Deformation tensor

Measures the change of metric in the deformed body

Strain Tensor



• Example : Rigid Body motion entails no deformation $\phi(X) = RX + T$

 $F(X) = \nabla \phi(X) = R \qquad C = R^T R = Id$

- Strain tensor captures the amount of deformation
 - It is defined as the "distance between C and the Identity matrix"

$$E = \frac{1}{2} \left(\nabla \phi^T \nabla \phi - Id \right) = \frac{1}{2} \left(C - Id \right)$$

Strain Tensor



- Diagonal Terms : E_i
 - Capture the length variation along the 3 axis



- Off-Diagonal Terms : γ_i
 - Capture the shear effect along the 3 axis



Analogy 1D-3D Elasticity



1D I	Elasticity	3D Elas	sticity
Deformat Gradier	tion $\frac{d\phi}{dX}$	Deformation Gradient	$\nabla \phi(X)$
Square St Ratio	retch $s^2 = \left(\frac{d\phi}{dX}\right)^2$	RCG-Deformation Tensor	ⁿ $C = \nabla \phi^T \nabla \phi$
Green Strain	$\varepsilon(s) = \frac{1}{2} \left(s^2 - 1 \right)$	Green Strain Tensor	$E = \frac{1}{2} \left(\nabla \phi^T \nabla \phi - Id \right)$
SVK Strain	$\lambda A(\varepsilon(s))^2$	SVK Strain	λ
Energy	$W(\Lambda) =$	Energy	$w(X) = \frac{\pi}{2} (tr E)^2 + \mu tr E^2$

Linearized Strain Tensor



Use displacement rather than deformation

$$\nabla \phi(X) = Id + \nabla U(X)$$
$$E = \frac{1}{2} \left(\nabla U + \nabla U^T + \nabla U^T \nabla U \right)$$

Assume small displacements

$$E_{Lin} = \frac{1}{2} \left(\nabla U + \nabla U^T \right)$$

Hyperelastic Energy



- The energy required to deform a body is a function of the invariants of strain tensor E :
 - Trace $E = I_1$
 - Trace $E^*E = I_2$
 - Determinant of $E = I_3$



$$W(\phi) = \int_{\Omega} w(I_1, I_2, I_3) dX$$

Total Elastic Energy
Linear Elasticity

Isotropic Energy

$$w(X) = \frac{\lambda}{2} (tr E_{Lin})^2 + \mu tr E_{Lin}^2$$

 (λ, μ) : Lamé coefficients

Hooke's Law

w(X) : density of elastic energy

- Advantage :
 - Quadratic function of displacement

$$w = \frac{\lambda}{2} (div U)^{2} + \mu \|\nabla U\|^{2} - \frac{\mu}{2} \|rot U\|^{2}$$

- Drawback :
 - Not invariant with respect to global rotation
- Extension for anisotropic materials



Shortcomings of linear elasticity



 Non valid for «large rotations and displacements»



St-Venant Kirchoff Elasticity



Isotropic Energy

$$w(X) = \frac{\lambda}{2} (tr E)^2 + \mu tr E^2$$

 (λ, μ) : Lamé coefficients

Advantage :

- Generalize linear elasticity
- Invariant to global rotations

Drawback :

- Poor behavior in compression
- Quartic function of displacement
- Extension for anisotropic materials

St Venant Kirchoff vs Linear Elasticity





Other Hyperelastic Material



Neo-Hookean Model

$$w(X) = \frac{\mu}{2}trE + f(I_3)$$

Fung Isotropic Model

$$w(X) = \frac{\mu}{2}e^{trE} + f(I_3)$$

Fung Anisotropic Model

$$w(X) = \frac{\mu}{2}e^{trE} + \frac{k_1}{k_2}\left(e^{k_2(I_4-1)} - 1\right) + f(I_3)$$

- Veronda-Westman $w(X) = c_1(e^{\gamma trE}) + c_2 trE^2 + f(I_3)$
- Mooney-Rivlin: $w(X) = c_{10}trE + c_{01}trE^2 + f(I_3)$

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- Biomechanical behavior of biological tissue is very complex
- Most biological tissue is composed of several components :
 - Fluids : water or blood
 - Fibrous materials : muscle fiber, neuronal fibers, ...
 - Membranes : interstitial tissue, Glisson capsule
 - Parenchyma : liver or brain

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In stress-strain relationships there are :



Parameter estimation

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- Complex for biological tissue :
 - Heterogeneous and anisotropic materials
 - Tissue behavior changes between in-vivo and in-vitro
 - Ethics clearance for performing experimental studies
 - Effect of preconditioning
 - Potential large variability across population

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- Different possible methods
 - In vitro rheology
 - In vivo rheology
 - Elastometry
 - Solving Inverse problems

- In vitro rheology
- can be performed in a laboratory. Technique is mature
- Not realistic for soft tissue (perfusion, ...)



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<u>.</u>

- can provide stress/strain relationships at several locations
- Influence of boundary conditions not well understood



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Elastometry (MR, Ultrasound)



- mesure property inside any organ non invasively
- validation ? Only for linear elastic materials



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Inverse Problems

approach)

• well-suited for surgery simulation (computational



• require the geometry before and after deformation





- Still difficult to find "reliable" soft tissue material parameters
- Example : Liver soft tissue characterization

First Author	Experimental Technique	Liver Origin	Young
			Modulus (kPa)
Yamashita [111]	Image-Based	Human	Not Available
Brown [15]	in- $vivo$	Porcine Liver	≈ 80
Carter [17]	in- $vivo$	Human Liver	≈ 170
Dan [27]	ex- $vivo$	Porcine Liver	≈ 10
Liu [62, 61]	ex- $vivo$	Bovine Liver	Not Available
Nava [76]	in-vivo	Porcine Liver	≈ 90
Miller [74]	in- $vivo$	Porcine Liver	Not Available
Sakuma [92]	ex- $vivo$	Bovine Liver	Not Available

Table 2: List of published articles providing some quantitative data about the biomechanical properties of the liver.

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Discretisation techniques

- Four main approaches :
 - Volumetric Mesh Based
 - Surface Mesh Based
 - Meshless
 - Particles

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Different types of meshes

Surface Elements :



Triangle

3, 12 nodes and more







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Structured vs Unstructured meshes

Example 1 : Liver meshed with hexahedra

3 months work (courtesy of ESI)



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Example 2: Liver meshed with tetrahedra

Automatically generated (1s)

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Volumetric Mesh Discretization

- Classical Approaches :
 - Finite Element Method (weak form)
 - Rayleigh Ritz Method (variational form)
 - Finite Volume Method (conservation eq.)
 - Finite Differences Method (strong form)
- FEM, RRM, FVM are equivalent when using linear elements

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- Step1 : Choose
 - Finite Element (e.g. linear tetrahedron)
 - Mesh discretizing the domain of computation
 - Hyperelastic Material with its parameters
 - Boundary Conditions



Step2

Write the elastic energy required to deform a single element



 $u(P_i) = Q_i - P_i = U_i$

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$$u(X) = \sum_{i=1}^{4} \lambda_i(X) u(P_i)$$

$$\nabla \lambda_i(X) = -\frac{M_i}{6V(T)}$$

$$trE = -\sum_{i} \frac{M_i \cdot U_i}{6V(T)}$$



Step3

Sum to get the total elastic energy

$$W(U) = \int_{\Omega_h} w(I_1, I_2, I_3) dX = \sum_{T_i} W_{T_i} = U^T K U$$

• Write the conservation of energy

SJ





Step3 Write first variation of the energy : Linear Elasticity KU = RStatic case $M\ddot{U} + C\dot{U} + KU = R(t)$ Dynamic case HyperElasticity=NonLinear Elasticity K(U) = RStatic case

 $M\ddot{U} + C\dot{U} + K(U) = R(t)$

Dynamic case

Surface-Based Methods



- Possible approaches :
 - Boundary Element Models (BEM)
 - Based on the Green Function of the linear elastic operator
 - Requires homogeneous material
 - Matrix Condensation
 - Full Matrix inversion
 - Iterative Precomputed Generation
 - Solve 3*Ns equations F=KU

Other Methods



Meshless Methods

- Use only points inside and specific shape functions
- Can better optimize location of DOFs
- Can cope with large deformations
- Deformation accuracy unknown

Particles

 Smooth Particles Hydrodynamics that interact based on a state equation

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Surgery Simulation



- Surgery Simulation must cope with several difficult technical issues :
 - Soft Tissue Deformation
 - Collision Detection
 - Collision Response
 - Haptics Rendering
- Real-time Constraints :
 - 25Hz for visual rendering
 - 300-1000 Hz for haptic rendering

Precomputed linear elastic model





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Tensor-Mass Models (low resolution)





N = 1394 (6342 Tétraèdres)

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Simulation of surgical gestures





Gliding



Gripping





Cutting (plan atomical Human project Cutting (US)



Hepatic Surgery Simulation



Fiber Tracking on the Average Cardiac



Electrophysiology Simulation







Normal heart Ectopic Pacing

Pseudo-potential Blue: excitable Red: depolarised Yellow: refractory Infarcted Area 10 times less conductive →Ventricular tachycardia →Ventricular fibrillation?



Electrophysiology Simulation





Normal heart Ectopic Pacing

Depolarisation Front Blue: depolarised side Red: excitable side Black: Repolarisation Front Infarcted Area 10 times less conductive →Ventricular tachycardia →Ventricular fibrillation?

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Cardiac Simulation



- 4 Cardiac Phases:
 - Filling
 - Isovolumetric Contraction
 - Ejection
 - Isovolumetric Relaxation

- 2 Volumetric Conditions:
 - Pressure Field in the endocardium
 - Isovolumetric Constraint of myocardium





Thank you for your attention